FULL FREQUENCY TRANSMISSION LOSS MODELLING USING FEM, BEM, SEA AND “FE/SEA COUPLED”

DENIS BLANCHET, ARNAUD CAILLET, WILLEM VAN HAL

ESI, Werner-Eckert-Str. 6, München, Germany, 81829
dbl@esigmbh.de, acl@esigmbh.de, wvh@esi-group.com

Transmission loss modelling plays a vital role in the design of acoustically efficient products. Whether in the aircraft, automotive or train industry, transmission loss measurements and simulation are used to improve the vibro-acoustic performance of components. Since fewer prototypes are built and therefore fewer tests can be made, the accurate prediction of transmission loss over the full frequency range of interest has become increasingly important. Modelling transmission loss of a complex structure involves the use of deterministic methods such as FEM (Finite Element Method) and BEM (Boundary Element Method) at lower frequencies and statistical methods such as SEA (Statistical Energy Analysis) at higher frequencies. Recently, a theoretical breakthrough in vibro-acoustics provided the ability to rigorously couple FEM and SEA in a single model. This paper introduces the theoretical foundation of the “FE/SEA coupled” approach and compares transmission loss predictions with traditional methods (FEM, BEM, SEA) and with test. It also compares computation time and memory usage since “FE/SEA Coupled” reduces the DOFs of the linear system to be solved.

Introduction

Rapid prototyping requires increased use of simulation since number of prototype is continuously reduced. Predictive simulation removes the need for physical prototypes until late in a design process. In many industries, transmission loss (TL) of components is a major vibro-acoustic performance indicator. For simulation to be of any use, it needs to cover the full frequency analysis. Furthermore, market pressure requires that TL computation be performed in the shortest of time to allow for optimization of components properties.

The objective of this paper is to present a new method for computing transmission loss of complex structures for full frequency analysis.

As an illustration of the method, two different honeycomb panels are studied. All numerical results presented in this paper are computed using the commercial vibro-acoustic software VA One [1] .

Numerical methods investigated

In the low frequency domain, the Finite Element Method (FEM) is well suited for structures and acoustic fluids where a low number of modes are present. It provides a good representation of the physics in a frequency range where boundary conditions (BC) has a non-negligible influence on the results. Boundary Element Method (BEM) is well suited for low frequency representation of fluid and is often combine to a FEM representation of the structure to compute TL. These methods are deterministic and usually computationally expensive but highly accurate.

In the high frequency domain, Statistical Energy Analysis (SEA) has been widely used for vibro-acoustic predictions on system and component studies. It is well suited to describe structures or acoustic fluids where many modes are present. This method is extensively used in aerospace and aircraft industry where honeycomb and other composite material are often used. It is also widely used in the automotive industry where TL of components can be optimized to improve performance of the sound package of a vehicle.

Both deterministic and statistical methods are using mode representation to build a model of the real system. In the FE method, the modes are represented by eigen frequency and eigen vectors and in the SEA method, modes are represented in terms of modal densities (Figure 1).

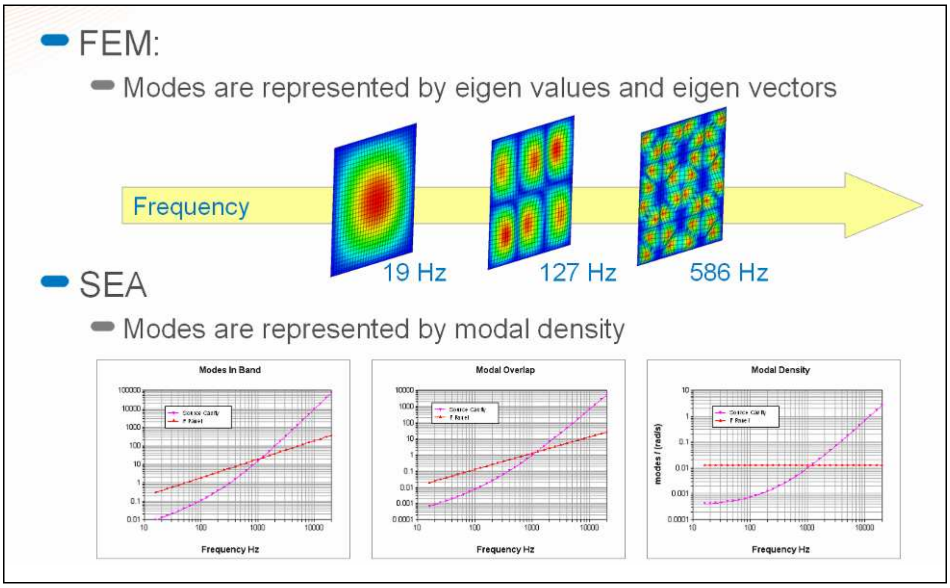

Figure 1: Representation of modes in FE and SEA methods

The full frequency analysis proposed in this paper is a combination of the deterministic FEM representation of the structure and the statistical SEA representation of the fluid in the large source and receiver room. The FEM and SEA content of a model are coupled together through the use of “FE/SEA Coupled” formulation (hybrid coupling). The structure is excited by a statistical Diffuse Acoustic Field (DAF) providing proper random character of the incident waves on the structure.

Introduction to Hybrid FE/SEA (From [2])

Hybrid FE/SEA method

A hybrid FE/SEA method ideally combines the low frequency performance of the FE method with the high frequency performance of SEA to produce a robust method that can be applied across the whole frequency range. However, the coupling of FE and SEA into a single model is difficult because the methods differ in two ways: (i) FE is based on dynamic equilibrium while SEA is based on the conservation of energy flow, and (ii) FE is a deterministic method while SEA is inherently statistical. Recently Shorter and Langley [4] have developed a new method of realising this coupling, which is based on wave concepts rather than the modal type of approach employed in reference [3] . At the heart of the method is a reciprocity result [5] regarding the forces exerted at the boundaries of an SEA subsystem. The method is briefly explained in the following paragraphs, It can be noted that references [4] and [5] contain a more formal and rigorous derivation of the hybrid method than that reported here.

In the mid-frequency range some components of a complex structure (for example thin panels) display short wavelength vibrations and are sensitive to the effects of random uncertainties, while others (for example beams) show little variation in their dynamic properties and are essentially deterministic. In the hybrid method proposed by Shorter and Langley [4] , the deterministic components are modelled by using the finite element method, while the random components are modelled as SEA subsystems.

A key feature of the method is the concept of a “direct field” or “power absorbing” dynamic stiffness matrix associated with each SEA subsystem. Consider for example a thin plate that is excited at the boundaries. The excitation generates waves that propagate through the plate and are reflected repeatedly at the boundaries; the total dynamic stiffness matrix of the plate, phrased in terms of the edge degrees of freedom, has contributions from all of these reflections. Suppose now that the response is viewed in two parts: 1) the contribution from the initial generated waves, prior to any boundary reflections. This can be called the “direct field”, 2) the contribution from waves produced on the first and all subsequent reflections. This can be called the “reverberant field”. The direct field dynamic stiffness matrix can be defined as that resulting from the presence of the direct field waves – this matrix corresponds to “power absorbing” behaviour, in the sense that the direct field waves all propagate energy away from the boundaries. Such a matrix can be found analytically for each of the subsystems by a variety of methods.

The Hybrid FE/SEA equations

The starting point for the hybrid method is to identify those parts of the system response that will be described by SEA subsystems. The remaining part of the system (which can be considered to be the “deterministic” part) is then modelled by using the FE method. For example, it might be decided that the bending motions of the panels of a structure have a short wavelength of deformation and will be described using SEA subsystems. The bending degrees of freedom of these panels will then be omitted from the FE model of the system, at all points other than the panel boundaries. The relevant “direct field” dynamic stiffness matrix is then added to the FE model at the panel boundaries, and this augmented FE model is then used in the subsequent analysis. If the degrees of freedom of the deterministic part are labelled q, then the governing equations of motion (for harmonic vibration of frequency ω) will have the form

$$D_{tot}q = f + \sum_k f_{rev}^{(k)}, \qquad D_{tot} = D_d + \sum_k D_{dir}^{(k)} \tag{1,2}$$

The summation is over the number of SEA subsystems in the model, D_{dir}^{k} and D_d represents the direct field dynamic stiffness matrix associated with subsystem k. Furthermore, D_d is the dynamic stiffness matrix given by the finite element model of the deterministic part of the system, f is the set of external forces applied to this part of the system, and represents the force arising from the reverberant field in subsystem k, which is not accounted for in D_{dir}^{k}. The matrix D_{tot} is the dynamic stiffness matrix of the FE model (excluding the SEA subsystem degrees of freedom), when augmented by the direct field dynamic stiffness matrix of each SEA subsystem. It should be noted that equations (1) and (2) are exact – all that has been done is to split the forces arising from the SEA subsystems into a direct field part, which is accounted for by D_{dir}^{k}, and a reverberant part which is carried to the right hand side of equation (1). The following result (Shorter and Langley [5]) is central to the development of the hybrid method:

$$S_{ff}^{(k),rev} \equiv E\left[f_{rev}^{(k)} f_{rev}^{(k)*T} \right] = \left(\frac{4E_k}{\omega \pi n_k} \right) \mathrm{Im}\left\{ D_{dir}^{(k)} \right\} \tag{3}$$

Here E_k and n_k are respectively the (ensemble average) vibrational energy and the modal density of the k^{th} subsystem. Equation (3) implies that the cross-spectral matrix of the force exerted by the reverberant field is proportional to the resistive part of the direct field dynamic stiffness matrix, which is a form of diffuse field reciprocity statement.

These basic equations can be combined and rewritten to lead to the following energy balance equation for subsystem j:

$$\omega(\eta_j + \eta_{d,j})E_j + \sum_k \omega \eta_{jk} n_j \left(E_j / n_j - E_k / n_k \right) = P_{in,j}^{ext} \tag{4}$$

And the cross-spectral matrix of the response q can be written as follows:

$$S_{qq} = D_{tot}^{-1} \left[S_{ff} + \sum_k \left(\frac{4E_k}{\omega \pi n_k} \right) \mathrm{Im}\left\{ D_{dir}^{(k)} \right\} \right] D_{tot}^{-1*T} \tag{5}$$

Equations (4) and (5) form the two main equations of the “Hybrid FE/SEA” method. It is clear that these equations couple FE and SEA methodologies: equation (4) has precisely the form of SEA, but the coupling loss factors η_{jk} and loss factors $\eta_{d,j}$ are calculated by using the FE model augmented by the direct field dynamic stiffness matrices; furthermore, equation (5) has the form of a standard deterministic FE analysis, but additional forces arise from the reverberant energies in the subsystems. If no SEA subsystems are included then the method becomes purely FE; on the other hand, if only the junctions between the SEA subsystems are modelled by FE, then the method becomes purely SEA, with a novel method of computing the coupling loss factors.

Numerical example

The numerical examples presented in this paper contain one flat honeycomb panel represented with FEM. Panel size is 1.40m by 1.15m. All honeycomb properties are listed in Table 1. These properties and the experimental results are taken from [6] .

Table 1: Properties of honeycomb panels

		Configuration 1	Configuration 2
Face Sheet (Isotropic Material)	Thickness (d)	5.842×10^{-4} m	4.572×10^{-4} m
	Density (ρ)	1716 kg/m^3	1778 kg/m^3
	Young's Modulus (E)	6.128×10^{10} Pa	1.523×10^{10} Pa
	Poisson's Ratio (ν)	0.143	0.142
	Loss factor (η)	0.05	0.05
Nomex Core (Orthotropic Material)	Thickness (d)	0.9017×10^{-2} m	1.905×10^{-2} m
	Density (ρ)	128.1 kg/m^3	48.1 kg/m^3
	Young's Modulus (E_{xx})	6.895×10^{5} Pa	6.895×10^{5} Pa
	Young's Modulus (E_{yy})	6.895×10^{5} Pa	6.895×10^{5} Pa
	Young's Modulus (E_{zz})	5.792×10^{8} Pa	1.310×10^{8} Pa
	Shear Modulus (G_{yz})	7.033×10^{7} Pa	2.550×10^{7} Pa
	Shear Modulus (G_{zx})	1.570×10^{8} Pa	4.900×10^{7} Pa
	Shear Modulus (G_{xy})	6.985×10^{5} Pa	6.985×10^{5} Pa
	Poisson's Ratio (ν_{yz})	0.01	0.01
	Poisson's Ratio (ν_{zx})	0.01	0.01
	Poisson's Ratio (ν_{xy})	0.50	0.50
	Loss factor (η)	0.05	0.05

FEM-BEM model

The panel is meshed using a minimum of 6 elements per wavelength. The honeycomb construction is represented using a PCOMP card. A frequency independent Damping Loss Factor (DLF) is used. To represent the large source and receiver rooms, BEM (Boundary Element Method) fluids are used. The BEM meshed have been coarsened and is valid to 1000Hz with 6 element per wavelength. A Diffuse Acoustic Field (DAF) is used to represent the panel excitation. This DAF is created using 50 plane waves (Figure 2)

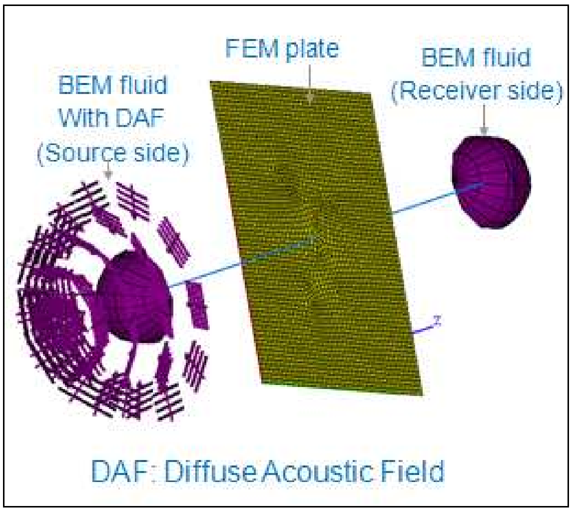

Figure 2: FEM-BEM Transmission Loss model.

“FE/SEA Coupled”model

The panel is meshed using a minimum of 6 elements per wavelength. The honeycomb construction is represented using a PCOMP card. A frequency independent Damping Loss Factor (DLF) is used. To represent the large source and receiver rooms, two Semi-Infinite Fluids (SIF) are used. These are free field propagation models (anechoic termination) providing the right acoustic impedance to the FEM honeycomb panel. A Diffuse Acoustic Field (DAF) is used to represent the panel excitation (Figure 3 right side model).

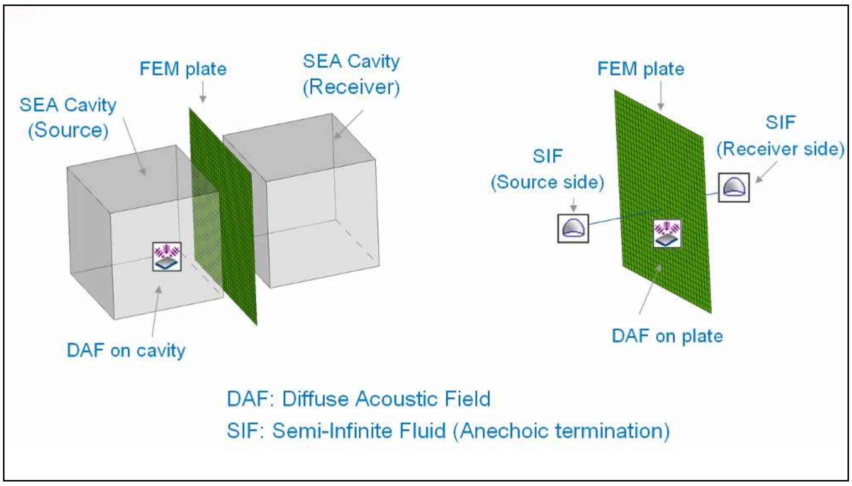

Figure 3: Equivalent Transmission Loss models. Left: DAF exciting a SEA cavity. Right: DAF directly exciting panel, fluids represented by SIF

SEA model

The plate is described using a SEA sandwich shell. The model contains two large acoustic SEA cavities and the source cavity has a pressure constraint.

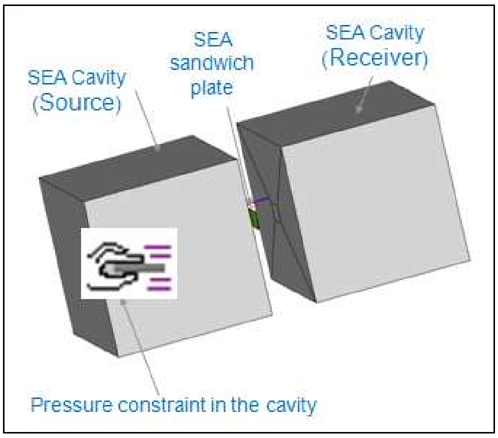

Figure 4: SEA Transmission Loss model.

Numerical models vs experimental results

In this work, the boundary conditions around the panel were assumed pinned. The correlation between the different numerical methods and experimental results are presented in Figure 5 and Figure 6. The deterministic method (FEM-BEM) provides accurate results but is computationally expensive and is limited to frequency range 10 to 1000 Hz. On the other hand, pure SEA is accurate from 400 Hz onward and is extremely cheap to compute. The “FE/SEA Coupled” method provides an accurate prediction of TL on the whole frequency domain (10 to 8000 Hz) and is relatively cheap to compute when compared with FEM-BEM approach.

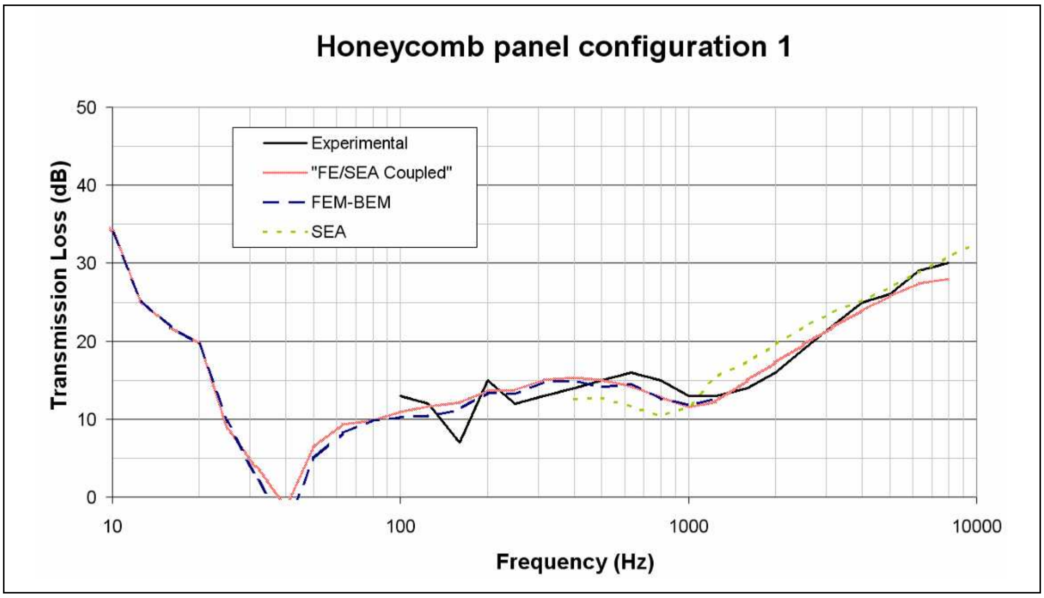

Figure 5: Comparison between numerical methods and experimental results. Configuration 1

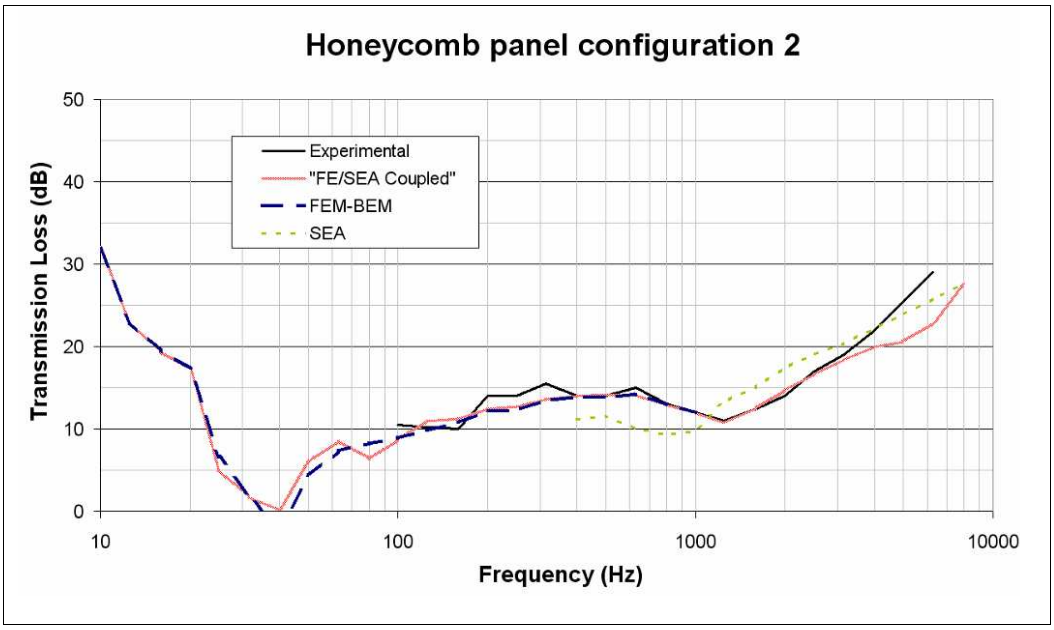

Figure 6: Comparison between numerical methods and experimental results. Configuration 2

Computation time

As was shown in previous section, "FE/SEA Coupled" method can cover the full frequency domain of interest allowing engineers to build and run a single model. This can be seen as a great advantage in model management. Furthermore, "FE/SEA Coupled" also brings other advantages such as computation time since this method is 10 times faster than the FEM-BEM method (Table 2). Note that in Table 2, the time indicated do not include the computation of structural models for FEM-BEM and "FE/SEA Coupled". Furthermore, "FE/SEA Coupled" has no limitation on the number of wetted nodes.

Table 2: Transmission Loss computation time for different methods

	10 to 1000 Hz (Δf = 10 Hz)	*10 to 8000 Hz (Δf = 10 Hz)*
FEM-BEM	50 min.	*
"FE/SEA Coupled"	5 min.	2 Hours
SEA**	-	3 sec.

** Not possible with BEM, number of wetted nodes too large*

*** 1/3 octave frequency band (valid from 400 Hz to 8kHz)*

Conclusion

This paper has introduced the new "FE/SEA Coupled" method as an efficient way of modelling transmission loss for full frequency analysis in a single model at a fraction of the computational and memory cost of FEM-BEM method. Two honeycomb panels transmission loss computations were compared with test and correlation can be characterized as excellent with deviation less than 3 dB in a broad frequency range.

The new method "FE/SEA Coupled" is a general method and is not limited to transmission loss problems of honeycomb flat panels. It can be used with more complex structures such as ribbed composite panels, curved panels. Also, trim (including porous media) can be added to any such panels using the widely known Transfer Matrix Method (TMM) or an explicit FEM poro-elastic formulation.

Finally, the "FE/SEA Coupled" method is well adapted to compute Transmission Loss of complex material. The model building effort is low since using FE for structural component (models usually available). It covers a wide frequency range, the accuracy is acceptable and the method is at least 10 times faster than FEM-BEM.

References

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